7. L. D. Landau and E. M. Lifshits, Continuum Mechanics [in Russian], GTII, Moscow (1944).
8. V. G. Levich, Physicochemical Hydrodynamics, Prentice-Hall (1962).
9. A. V. Lykov (ed.), Electrorheological Effect [in Russian], Nauka i Tekhnika, Minsk (1972).

EXPOSURE OF THE FLUX OF A MEDIUM TO A REGULAR SYSTEM OF LIGHT SOURCES

Yu. A. Kashin, G. Yu. Kashina,
and L. G. Kravchenko

UDC 541.14:533

An analysis is performed of the stationary distribution of the specific dosage of radiation energy absorbed by a medium from a regular system of tubular light sources submerged in an infinite flux of the medium.

One of the fundamental characteristics in the photochemical and radiation kinetics of molecular and biological systems simulated in the form of continuous media is the specific dosage of the radiation energy absorbed by the medium (exposure dose) which is determined by the relationship

$$q = \lim_{\Delta V \to 0} \Delta E / \Delta V,$$

where ΔE is the quantity of radiation energy absorbed by a volume element of the medium. In conformity with the definition, the local value of the exposure dose $q(\mathbf{r}, t)$ for volume elements of the flux being exposed continuously in the space of a fixed domain Ω bounded by the surface dΩ is found from solution of the problem

$$\frac{\partial q}{\partial t} + \mathbf{v} \cdot \nabla q = kI, \tag{1}$$

$$q(\mathbf{r}, 0) = \varphi_1(\mathbf{r}) \text{ on } \Omega, \tag{2}$$

$$q(\mathbf{r}, t) = \varphi_2(\mathbf{r}, t) \text{ in part, where } \mathbf{n} \cdot \mathbf{v} \leqslant 0. \tag{3}$$

Solutions of this problem are known for one-dimensional or axisymmetric stationary processes proceeding in tubular photoreactors with a single light source [1-4]. The process of exposing a flux of absorbing medium by a regular system of tubular lamps similar to the process of exposing a water stream in casette bactericidal apparatus of the type OV-PK-RKS [5] is analyzed in this paper. Since the presence of light-absorbing impurities in water results in attenuation of the light flux, it is then interesting to estimate the influence of such impurities on the exposure efficiency, and also to investigate the optimization condition for this process.

Let us consider the following steady-state two-dimensional process. An unlimited flux of medium reaches a certain system of identical tubular light sources whose axes are mutually parallel and perpendicular to the vector $\mathbf{v}_0$ of a flux of particles at infinity. We consider this system of light sources regular in the sense that adjacent projections of the lamp axes on the plane perpendicular to $\mathbf{v}_0$ are separated by the same distance 2b which we call the lattice spacing of the lamps. Assuming the interaxial spacing c between any adjacent lamps to be considerably greater than their radius a ($c \gg a$), we can assume that the perturbations in medium flux by adjacent lamps exert no substantial influence on the hydrodynamic situation

Gomel State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 48, No. 3, pp. 437-442, March, 1985. Original article submitted December 20, 1983.

 0022-0841/85/4803-0322$09.50

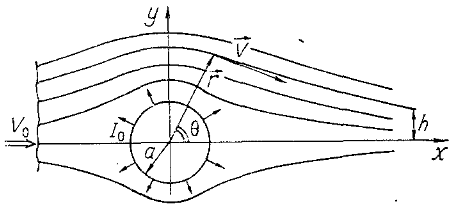

Fig. 1. Fluid flux reaching a lamp.

near a given lamp. Consequently, the result of action of the whole system of light sources can be considered as the superposition of independent actions of each of the sources of this system.

To analyze the exposure of the flux to a single lamp (Fig. 1), we use the following hypotheses and constraints. We assume light absorption by the medium to satisfy the Bouger–Lambert–Beer law

$$I = I_0 \frac{a}{r} \exp[-k(r-a)].$$

We assume the absorption coefficient k constant. We limit ourselves to the case of practical interest of large values of the Reynolds number Re = $2av_0/\nu$ and we assume that the depth of light penetration, equal to 1/k, will significantly exceed the thickness of the stagnation hydrodynamic boundary layer δ of the lamp

$$1/k \gg \delta = a(\mathrm{Re})^{-1/2}.$$

Then in conformity with known methods of hydrodynamics [6], the existence of the boundary layer can be neglected and the known Euler solution for the problem of ideal fluid flow around a cylinder can be taken for the velocity field **v**(r). According to this solution, the components v_r, v_θ of the velocity vector **v** in a r, θ polar coordinate system are expressed by using the stream function

$$\Psi = v_0 (r - a^2/r) \sin\theta \tag{4}$$

by the dependence

$$v_r = \frac{1}{r}\frac{\partial \Psi}{\partial \theta} = v_0 \left(1 - \frac{a^2}{r^2}\right)\cos\theta,$$

$$v_\theta = -\frac{\partial \Psi}{\partial r} = -v_0 \left(1 + \frac{a^2}{r^2}\right)\sin\theta,$$

Under these conditions, by using the notation

$$Q = \frac{q}{q_0}, \quad q_0 = \frac{I_0}{v_0}, \quad R = \frac{r}{a}, \quad \varkappa = ak, \tag{5}$$

we convert (1) to the form

$$\left(R - \frac{1}{R}\right)\cos\theta \frac{\partial Q}{\partial R} - \left(1 + \frac{1}{R^2}\right)\sin\theta \frac{\partial Q}{\partial \theta} = \varkappa \exp[-\varkappa(R-1)]. \tag{6}$$

To integrate it, we form the system of ordinary differential equations of the characteristics [7]:

$$\frac{dR}{\left(R - \frac{1}{R}\right)\cos\theta} = \frac{d\theta}{-\left(1 + \frac{1}{R^2}\right)\sin\theta} = \frac{dQ}{\varkappa \exp[-\varkappa(R-1)]}.$$

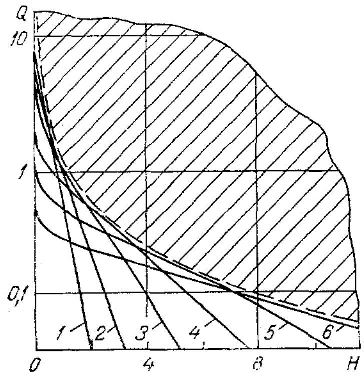

Fig. 2. Distribution of the specific absorbed radiation dosage Q from a single lamp over the stream width H for different optical density indices of the medium $\varkappa$: 1) 3.2; 2) 1.6; 3) 0.8; 4) 0.4; 5) 0.2; 6) 0.1.

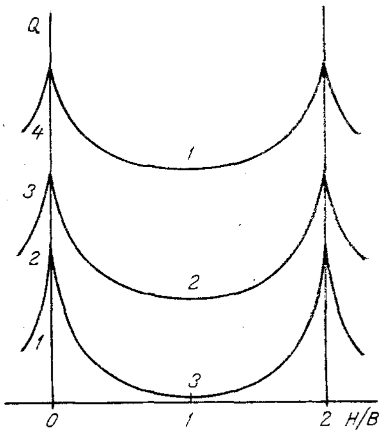

Fig. 3. Distribution of the total absorbed radiation dosage Q from 10 lamps for $\varkappa=1$ for a different lattice spacing B: 1) 0.5; 2) 1; 3) 4.

from whose solution we find that one of the families of characteristics is the curves

$$\left(R - \frac{1}{R}\right)\sin\theta = H = \text{const} \tag{7}$$

that coincide with the streamlines determined by the function (4). Noting that $R\sin\theta \to H = h/a$ as $R \to \infty$, we establish the physical meaning of the parameter H: it is numerically equal to the dimensionless spacing between this streamline and the trace of the lamp axis in the infinitely remote domain of the flux. We use the formulas

$$\sigma = R\sin\theta,\ \ \tau = (\sin\theta)/R \tag{8}$$

to introduce new variables which are related on the streamlines, in conformity with (7), by the equation

$$\sigma - \tau = H. \tag{9}$$

In these variables, for $H \neq 0$ and $\theta \in [0, \pi/2]$ Eq. (6) is converted to the form

$$\frac{dQ}{d\tau} = \Phi(\tau) \equiv -\frac{\varkappa \exp[-\varkappa(\sqrt{(\tau + H)/\tau} - 1)]}{4\tau\sqrt{1 - \tau(\tau + H)}} \tag{10}$$

along the streamline. To integrate this equality we assume that the initial medium is unexposed, we use the property of streamline symmetry relative to the vertical $\theta = \pi/2$ and we take into account that in conformity with (7) and (8), for $\theta \to 0$, $\tau \to 0$, while for $\theta = \pi/2$, $\tau \to T(H) = 2/(H + \sqrt{4 + H^2})$. Then

$$Q(H, \varkappa) = 2\int_T^0 \Phi(\tau)\, d\tau = \frac{\varkappa \exp \varkappa}{2} \int_0^T \frac{\exp\left(-\varkappa \sqrt{(\tau + H)/\tau}\right)}{\tau \sqrt{1 - \tau(\tau + H)}}\, d\tau. \tag{11}$$

For confirmation it is easy to see that the relationship (11) determines the function $F(Q, H, \varkappa) = 0$ single-valuedly. Graphs of this one-parameter family of curves, constructed on the basis of numerical computations, are shown in Fig. 2. It follows from the figure that the specific exposure dosage that reaches unlimited values on the stream axis will diminish rapidly with removal of the streamline from the axis and with growth of the index $\varkappa$ of the optical density of the medium. Media with higher optical densities here absorb a larger radiation dose near the lamp, and on the other hand, a lower dose far from the lamp.

The envelope of this family of curves $Q = f(H)$ which is determined by the solution of the system

$$F(Q, H, \varkappa) = 0, \ \partial F(Q, H, \varkappa)/\partial \varkappa = 0$$

as is known [8], and which is displayed by dashed lines in Fig. 2, is the curve for the ultimate exposure dosage. It separates the H, Q plane into two domains. The upper domain $Q > f(H)$, shaded in Fig. 2, is the domain of the unachievable dosage. If this element of the medium experienced exposure by a streamline removed a spacing H from the stream axis, while the effect required for this medium was achieved for $Q = Q_*$ and if $Q_* > f(H)$, then independently of the optical density of the medium, the element under consideration will obtain a dosage Q below Q_*: $Q < Q_*$.

Upon stream exposure of the regular system of equivalent lamps under consideration, this streamline will be removed from the traces of the lamp axes in the infinitely remote stream domain by the respective spacings

$$H,\ 2B - H,\ 2B + H,\ 4B - H,\ \ldots\ 2(m-1)B + H,\ 2mB - H,\ \ldots, \tag{12}$$

where $H \in [0, B]$. Substituting the values of the terms in the sequence (12) into (11) instead of H, we obtain partial values of the radiation dosage absorbed by this element of the medium from the appropriate lamp. Their sum is

$$Q = \frac{\varkappa e^{\varkappa}}{2} \sum_{m=1}^{\infty} \left\{ \int_0^{T_1(m)} \frac{\exp\left[-\varkappa \sqrt{\dfrac{\tau + 2(m-1)B + H}{\tau}}\right]}{\tau \sqrt{1 - \tau[\tau + 2(m-1)B + H]}}\, d\tau + \int_0^{T_2(m)} \frac{\exp\left(-\varkappa \sqrt{\dfrac{\tau + 2mB - H}{\tau}}\right)}{\tau \sqrt{1 - \tau(\tau + 2mB - H)}}\, d\tau \right\}, \tag{13}$$

where

$$T_1(m) = \frac{2}{2(m-1)B + H + \sqrt{4 + [2(m-1)B + H]^2}};$$

$$T_2(m) = \frac{2}{2mB - H + \sqrt{4 + (2mB - H)^2}},$$

determine the total exposure dosage for an element of the medium as a function of the spatial location of this streamline, the light sources distribution density, and the optical density of the medium.

The nature of this dependence can be traced from graphs of the function $Q = Q(H)$ computed by means of (13) and displayed in Fig. 3. As should have been expected, this function is spatially periodic, reaching a minimal value for $H = B$

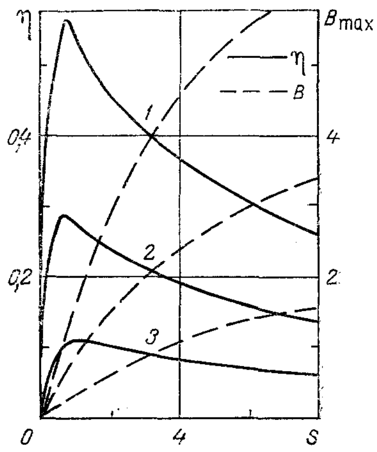

Fig. 4. Dependence of the efficiency η of the process and the maximum lamp array spacing B_{max} on the parameter S for $\varkappa=0.3$ and different values of λ: 1) 0; 2) 1; 3) 4.

$$P = Q_{\min} = \varkappa e^{\varkappa} \sum_{m=1}^{\infty} \int_0^{T_*} \frac{\exp[-\varkappa\sqrt{[\tau+(2m-1)B]/\tau}]}{\tau\sqrt{1-\tau[\tau+(2m-1)B]}}\, d\tau, \tag{14}$$

where

$$T_*(m, B) = \frac{2}{(2m-1)B+\sqrt{4+(2m-1)^2B^2}}, \quad m = 1, 2.$$

The value of $P(B, \varkappa)$ diminishes as B and $\varkappa$ increase, however the physical nature of the influence of each of these parameters is distinct. If an increase in the lamp lattice spacing B is accompanied by a general diminution in the energy expended in exposure of the stream, then an increase in the optical density index $\varkappa$ for a given medium can be related primarily to the presence of light-absorbing impurities in the medium, which play the part of an internal filter of the system. Consequently, an effective action for this process will not produce all the absorbed energy but only that part which has been adsorbed by the medium itself and equals k_0/k.

Being guided by the above, we discuss selection of the parameters of the apparatus under consideration. We assume that the minimally allowable dose absorbed by the medium itself equals q* according to the conditions of the process, and we let

$$q_0 = Sq^*, \ \varkappa_0 = ak_0, \ \lambda = k_0/k - 1.$$

Then to assure exposure of the medium itself by a dose not less than q* it is sufficient that the following condition be satisfied

$$\frac{Pq_0}{1+\lambda} = q^* \quad \text{or} \quad \frac{PS}{1+\lambda} = 1.$$

Substituting the value of P from (14), we represent this condition in the form

$$1 = \varkappa_0 S \exp[\varkappa_0(1+\lambda)] \sum_{m=1}^{\infty} \int_0^{T_*} \frac{\exp[-\varkappa_0(1+\lambda)\sqrt{[\tau+(2m-1)B]/\tau}]}{\tau\sqrt{1-\tau[\tau+(2m-1)B]}}\, d\tau. \tag{15}$$

Considering $\varkappa_0$ a fixed parameter of the system, this relationship can be considered as an equation implicitly determining the maximum lamp lattice spacing B_{max} that will assure an exposure dose for the medium that is not below a given value dependent on the parameter λ governing the optical density of the mixture, and on the parameter $S \equiv q_0/q^* = I_0/v_0 q^*$ dependent on the stream velocity, the intensity of the lamp surface luminescence and q^*: $B_{max} = \varphi(S, \lambda)$.

Graphs of this function are shown by dashed curves in Fig. 4 for $\varkappa_0 = 0.3$ (which corresponds to exposure of water with $k = 20\ m^{-1}$ to the lamp DRT 2500). The function B_{max} grows as S grows and λ diminishes, i.e., the maximal lamp array spacing will be greater, the smaller the exposure dosage needed to achieve a given effect, and the purer the medium. Represented in this same figure are graphs of the dependence of the efficiency $\eta(S)$ defined as the ratio between the minimally needed exposure dosage of the flux and its mean exposure dosage:

$$\eta = \frac{B_{max} q^* v_0}{\pi a I_0} = \frac{1}{\pi S} \varphi(S, \lambda).$$

As follows from Fig. 4, the quantity η has a maximum in the domain S = 0.6-1. The curves 1 corresponding to the process of exposure of an ideally pure fluid (λ = 0) determine the maximum values of these functions while domains of their unachievable values lie above the curves 1. For instance, if the parameter S of a given exposure process equals 4, then the spacing between the projections of the lamp axes should not exceed B = 4.6, or 14 cm, while the process efficiency will not exceed 0.36 for any fluid.

NOTATION

a, lamp radius; b, half-spacing between nearest projections of the lamp axes in a plane perpendicular to the velocity vector **V**; $B = b/a$; $H = h/a$; h, spacing between streamlines and the trace of the lamp axes in the infinitely remote flow domain; I(r, t), scalar radiation intensity field; I_0, radiation intensity of the lamp surface; k, extinction coefficient of the medium; k_0, extinction coefficient of a medium without impurities m, the lamp number; **n**, external normal to the surface $d\Omega$; q, specific exposure dosage of the medium; $q_0 = I_0/v_0$; q^*, minimal dosage to achieve the needed effect; $Q = q/q_0$; r, θ, polar coordinate system; $R = r/a$, $S = q_0/q^*$; **V**(r, t), velocity field; v_0, stream particle velocity vector in the infinitely remote domain of the flow; v_r, v_θ, components of the velocity vector v; η, efficiency; $\varkappa = ak$; $\lambda = k/k_0 - 1$; ν , kinematic viscosity coefficient; $\pi = 3{,}14...$; $\sigma = R \sin\theta$; $\tau = \sin\theta/R$; Ψ , stream function

LITERATURE CITED

1. A. E. Cassano and J. M. Smith, "Photochlorination in a tubular reactor," AIChEJ, 12, No. 6, 1124-1133 (1966).
2. Takeshi Matsuura and J. M. Smith, "Light distribution in cylindrical photoreactors," AIChEJ, 16, No. 2, 321-324 (1970).
3. G. Spadoni, C. Stramigioli, and F. Santarelli, "Rigorous and simplified approach to the modelling of continuous photoreactors," Chem. Eng. Sci., 35, No. 4, 925-931 (1980).
4. A. Tournier, X. Deglise, J. C. Andre, and M. Niclause, "Experimental determination of the light distribution in a photochemical reactor: influence of the concentration of an absorbing substance," AIChEJ, 28, No. 1, 156-166 (1982).
5. Handbook on the Properties, Methods of Analysis and Purification of Water. Pt. 2 [in Russian], Naukova Dumka, Kiev (1980).
6. L. G. Loitsyanskii, Mechanics of Fluids and Gases [in Russian], Nauka, Moscow (1973).
7. N. M. Gyunter, Integration of First-Order Partial Differential Equations [in Russian], ONTI, GTTI, Leningrad—Moscow (1934).
8. G. A. Korn and T. M. Korn, Manual of Mathematics, McGraw-Hill (1967).